

Dynamics of encrypted information in the presence of imperfect operations

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Abstract

The original dense coding protocol is achieved via quantum channel generated between a single Cooper pair and a cavity. The dynamics of the coded and decoded information are investigated for different values of the channel's parameters. The efficiency of this channel for coding and decoding information depends on the initial state settings of the Cooper pair. It is shown that, these information increase as the detuning parameter increases or the number of photons inside the cavity decreases. The coded and decoded information increase as the ratio of the capacities between the box and gate decreases. In the presence of imperfect operation, the sensitivity of the information to the phase error is much larger than the bit flip error.

Keywords: Dense coding, Channels, Local and non-local information.

1 Introduction

One of the most obstacles of the quantum information processing is sending quantum and classical information safely. So, studying the behavior of information in a noise circumstance is one of most important tasks in the context of quantum communication [1]. Quantum dense coding, which was first proposed by Bennett and Wiesner[2], enables the communication of two bits of classical information with the transmission of qubit. Therefore it has an important applications in secure quantum communication. To achieve a perfect quantum dense coding one needs maximum entangled channels and perfect local operations which are difficult to be performed on the real world [3].

Therefore there are some efforts have been done to investigate the possibility of performing the dense coding protocol in imperfect circumstances. For example, overcoming a limitation of deterministic dense coding with a non-maximally entangled initial state has been investigated by Bourdon and Gerjuoy [4]. The dynamics and the robustness of the coded information over the noise Bloch channels are investigated in [5]. Xi-Han Li and et. al.[6] have used the idea of quantum dense coding to investigate the robust quantum key distribution protocols against two kinds of collective noise.

In this contribution, we discuss the dynamics of the coded and decoded information when the users apply imperfect operations during the coding process. Due to their potential in quantum information processing we use the generated entangled state between a Cooper pair and a cavity mode as a quantum channel [7, 8]. Metwally and et.al.[9] have investigated the entangled properties of this state and discussed the possibility of using it as quantum channel for quantum teleportation .

The paper is organized as follows: In Sec.2, an analytical solution of the suggested model is introduced. The coding protocol is performed, where the local operations are archived perfectly in Sec.3. In Sec.4, the dynamics of the coded and decoded information in the presence of two different types of noise are investigated. Due to this noise operations we quantify the disturbance of the decoded information in Sec.5. Finally, Sec.6 is devoted to discuss the results.

2 The Model

This model consists of subconducting box with a low-capacitance, C_J and Josephson junction, E_J biased by a classical voltage, V_g through a gate capacitance, C_g placed inside a single mode microwave cavity [10, 11, 12]. If the gate capacitance is screened from the quantized radiation field, then junction-field Hamiltonian, in the interaction picture can be written as,

$$\mathcal{H}_c = 4E_c(n - n_g)^2 - E_J \cos \phi, \quad (1)$$

where, $E_c = \frac{1}{2}e^2(C_J + C_g)$ is the charging energy, $E_J = \frac{1}{2}\frac{\hbar}{e}I_c$ is the Josephson coupling energy, e is the charge of the electron, $n_g = \frac{1}{2}\frac{V_g}{e}C_g$ is the dimensionless gate charge, n is the number operators of excess Cooper pair on the island and ϕ is the phase operator [12, 13]. If the Josephson coupling energy, E_J is much smaller than the charging energy i.e $E_J \ll E_c$, and the temperature is low enough, then the Humiliation (1) becomes

$$\mathcal{H}_c = -\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x, \quad (2)$$

where $B_z = -(2n - 1)E_{cl}$, E_{cl} is the electric energy, $B_x = E_J$ and $\sigma_x, \sigma_y, \sigma_z$ are Pauli matrices. This Cooper pair can be viewed as an atom with large dipole moment coupled to microwave frequency photons in a quasi-one-dimensional transmission line cavity (a coplanar waveguide resonator). The combined Hamiltonian for the Cooper qubit and transmission line cavity is given by [14, 15],

$$\mathcal{H} = \varpi a^\dagger a + \varpi_c \sigma_z - \lambda(\mu - \nu \sigma_z) + \sqrt{1 - \nu^2} \sigma_x (a^\dagger + a), \quad (3)$$

where, ω is the cavity resonance frequency, $\omega_c = \sqrt{E_J^2 + 16E_c^2(2n_g - 1)^2}$ is the transition frequency of the Cooper qubit, $\lambda = \frac{\sqrt{C_J}}{C_g + C_J} \sqrt{\frac{1}{2}\frac{\varpi}{\hbar}e^2}$ is coupling strength of resonator to the Cooper qubit, $\mu = 1 - n_g$, $\nu = \cos \theta$ and $\theta = -\arctan\left(\frac{1}{E_c}\frac{E_J}{2n_g - 1}\right)$ is mixing angle.

Assume that the system is initially prepared in the state $|\psi_s(0)\rangle = |\psi_c\rangle \otimes |n\rangle$, where $|\psi_c\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ is the initial state of the Cooper qubit, while $|n\rangle$ represents the initial state of the field. The time evolution of the initial system is given by,

$$\begin{aligned} |\psi_s(t)\rangle &= e^{-i\mathcal{H}t}|\psi_s(0)\rangle, \\ &= c_1|n, e\rangle + c_2|n - 1, e\rangle + c_3|n, g\rangle + c_4|n + 1, g\rangle, \end{aligned} \quad (4)$$

where,

$$\begin{aligned} c_1 &= \cos \gamma_{n+1}\tau - \frac{i\delta}{\gamma_{n+1}} \sin \gamma_{n+1}\tau, & c_2 &= i\frac{\delta}{\gamma_{n+1}}\sqrt{n-1} \sin \gamma_{n+1}\tau, \\ c_3 &= \cos \mu_n\tau + i\frac{\delta}{\mu_n} \sin \mu_n\tau, & c_4 &= -i\frac{\delta}{\gamma_{n+1}}\sqrt{n+1} \sin \gamma_{n+1}\tau, \end{aligned} \quad (5)$$

and $\delta = \frac{\Delta}{2\lambda}$, $\Delta = E_J - \varpi$ is the detuning parameter between the Josephson energy and the cavity field frequency, $\gamma_n = \sqrt{\delta^2 + n}$ and $\tau = \lambda t$ is the scaled time.

The entangled and separable behavior of this system are investigated in [9], where different initial states settings are consider. The effect of the Cooper pair and the cavity's parameters on these phenomena is discussed. Also, the possibility of using the generated entangled state between the field and Cooper pair to achieve quantum teleportation is studied [9]. In this context, we sheet the light on the dynamics of the coded and decoded information via this type of entangled states. Moreover, we investigate these phenomena when the used local operations are imperfect.

3 Quantum coding via perfect operation

For the sake of simplicity, we consider that the initial state of the system is $|\psi_s(0)\rangle = |n, e\rangle$. For this initial state setting, the final state of the system is given by,

$$|\psi_s(t)\rangle = c_1|n, e\rangle + c_4|n+1, g\rangle, \quad (6)$$

where c_1 and c_4 are given from (5). This state represents the entangled channel between the users Alice and Bob. To perform quantum coding protocol, the users need to apply a group of local operations. Due to the noise, these operations can not be performed perfectly. Therefore, our aim in this section is investigating the dynamics of the coded and decoded information when the operations are archived correctly. To show this idea, we implement the original dense coding protocol which has been proposed by Bennett and Wienser [2]. This protocol is described as follows:

1. Alice encodes two classical bits by using one of local unitary operations. If Alice applies these unitary operations randomly with probability p_i , then she codes the information in the state,

$$\rho_c = \sum_{i=0}^3 \left\{ p_i \sigma_{1i} \otimes I_2 \rho_s(t) \sigma_{1i} \otimes I_2 \right\}, \quad (7)$$

where,

$$\rho_s(t) = c_1^2|e, n\rangle\langle e, n| + c_1c_4^*|e, n\rangle\langle g, n+1| + c_4c_1^*|g, n+1\rangle\langle n, e| + c_4^2|g, n+1\rangle\langle g, n+1|, \quad (8)$$

represents the quantum channel between the users Alice and Bob and $\sigma_{1i} = I_1, \sigma_{1x}, \sigma_{1y}$ and σ_{1z} , are the unitary operators, which represent the local operations for Alice's qubit and I_2 is the identity operator for Bob's qubit. The coded information is given by ,

$$I_{cod} = -c_1^2 \log_2 c_1^2 - c_4^2 \log_2 c_4^2, \quad (9)$$

where it is assumed that Alice performs the local operations with an equal probability i.e., $p_i = \frac{1}{4}, j = 0...3$.

2. Alice sends her qubit to Bob, who makes joint measurements on the two qubits. The maximum amount of information which Bob can extract from Alice's message is bounded by,

$$I_{Bob} = \mathcal{S}\left(\sum_{j=0}^{j=3} p_j \rho(t)\right) - \sum_{i=0}^{i=3} p_i \mathcal{S}(\rho(t)), \quad (10)$$

where $\mathcal{S}(\cdot)$ is the von Neumann Entropy.

The dynamics of the coded information, I_{cod} and the decoded information, I_{Bob} are investigated in Fig.(1) for different values of the detuning parameter, where we fixed the values of other parameters. For small values of Δ , the coded information increases gradually to reach its maximum values and then decreases smoothly. As one increases Δ , the number of oscillations decreases and I_{cod} remain maximum for a longer time. This is clear by comparing the dot curve where we set $\Delta = 0$ and the dash-dot curve ($\Delta = 1$) in Fig.(1a). The effect of the detuning parameter on the decoded information is displayed in Fig.(1b). In general the amount of information which Bob can extract from the sending information increases as one

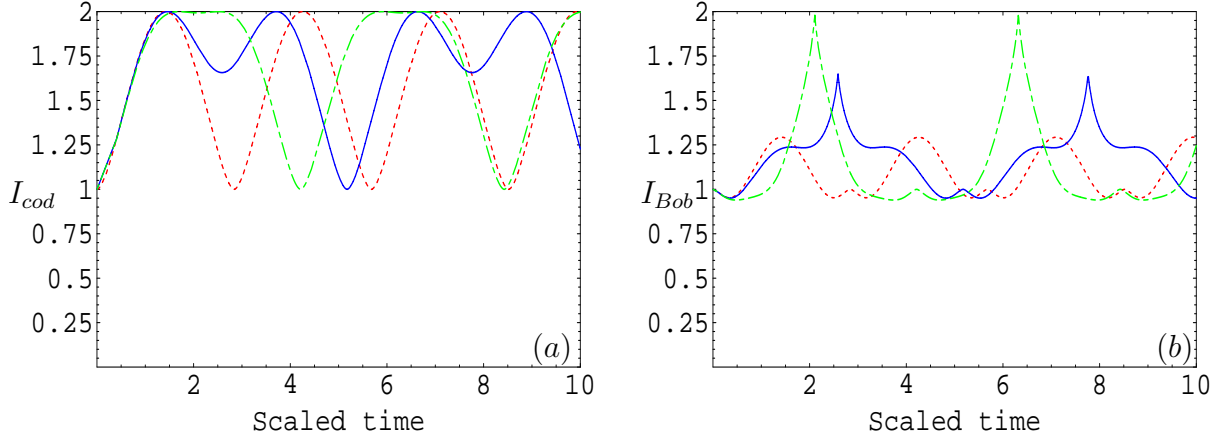


Figure 1: The dynamics of the information in the presence of perfect local operations where $n = 2$, the ratio $\kappa = \frac{\sqrt{C_j}}{C_g + C_j} = \frac{5}{2}$ and $\Delta = 0, 0.5, 1$ for the dot, solid and dash-dot curves respectively (a) coded information, I_{cod} (b) decoded information, I_{Bob} .

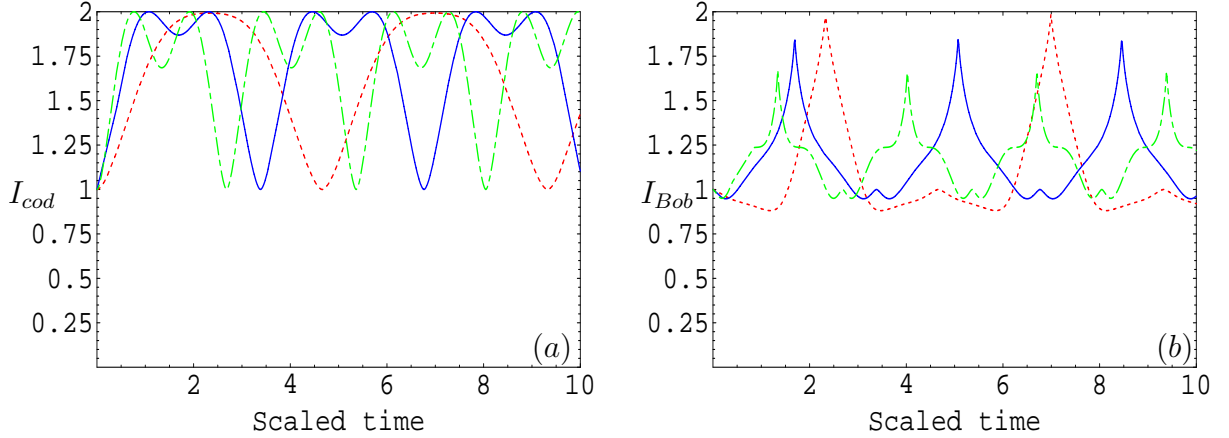


Figure 2: The same as Fig.(1), but $n = 1, 5$ and 10 for the dot, solid, dash-dot curves respectively and $\Delta = 1$.

increases the detuning parameter. However, the maximum extracted information depends on the value of Δ .

In Fig.(2), we investigate the effect of the number of photons, n on the dynamics of the coded and decoded information. Fig.(2a) shows the behavior of I_{cod} for different values of n while the detuning parameter, $\Delta = 1$. The general behavior of I_{cod} is the same as that depicted in Fig.(1a). However the number of oscillations increases as one increases n . The dynamics of the decoded information is displayed in Fig.(2b), where the maximum extracted information increases as one decreases the number of photons inside the cavity.

In this context, it is important to investigate the effect of the parameter κ , which represents the ratio between the capacities of the Cooper pair box and the gate. The dynamics of the coded and decoded information for different values of κ are shown in Fig.(3). It is clear that as κ decreases, the lower bound of coded information shifted to the right. This means that the coded information remains maximum for larger interval of time. This behavior is clearly seen by comparing Fig.(1a) and Fig.(3a). However as shown in Fig.(3b), for smaller

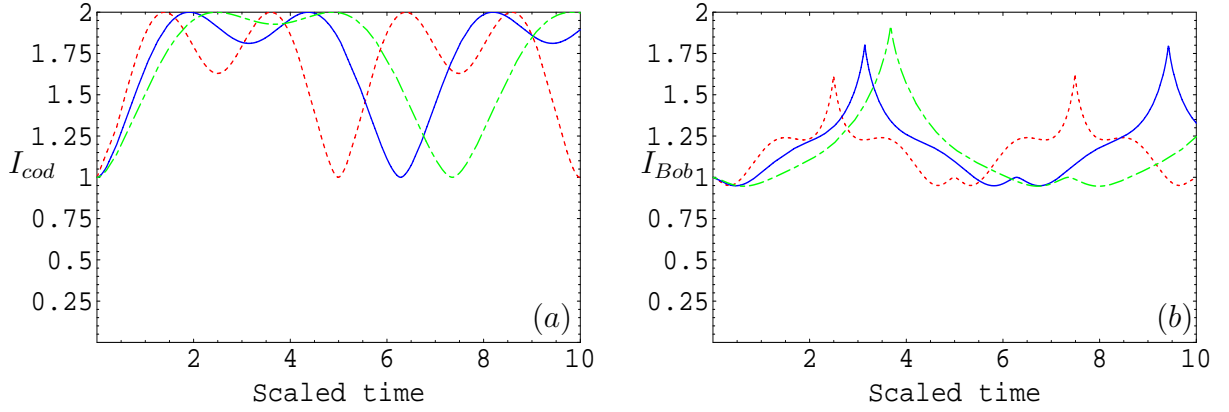


Figure 3: The same as Fig.(1), but the ratio, $\kappa = \frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ for the dot, solid and dash dot curves respectively, with $\Delta = 0.5$ and $n = 2$.

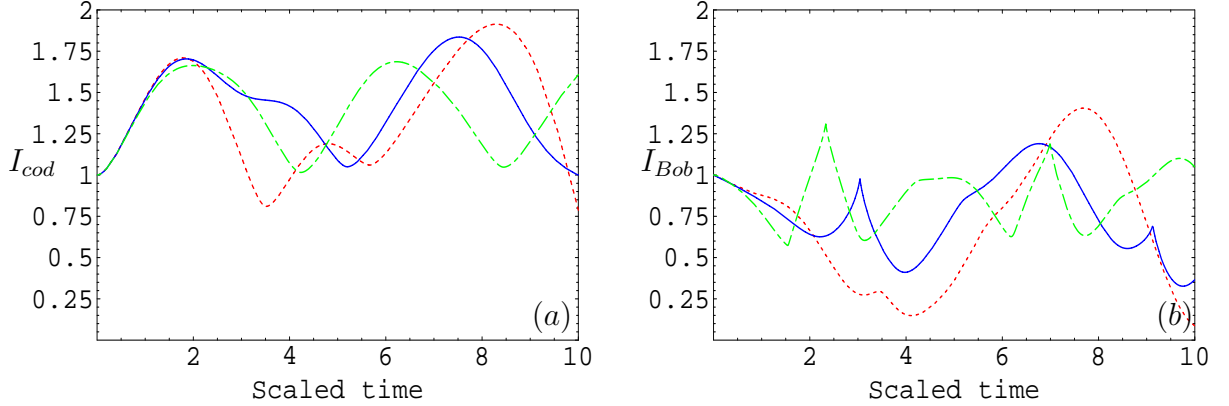


Figure 4: The same as Fig.(1), but the system is initially prepared in the state $\psi_s(0) = |n, g\rangle$.

values of κ , the maximum values of the decoded information increases and shifted to the right.

Let us end this section by considering another initial state settings of the system. Assume that the Cooper qubit is prepared initially in the ground state $|g\rangle$. Then the initial state of the total system is $\rho_s(0) = |n, g\rangle\langle n, g|$. The dynamics evolution of this system is given by

$$\rho_s(t) = |c_2|^2 |n-1, e\rangle\langle n-1, e| + c_2 c_3^* |n-1\rangle\langle n, g| + c_3 c_2^* |n, g\rangle\langle n-1, e| + |c_3|^2 |n, g\rangle\langle n, g|, \quad (11)$$

where c_2 and c_3 are given from (5). The users Alice and Bob use the state(11) as a quantum channel to perform the original dense coding protocol [2].

Fig.(4) describes the dynamics of the coded and decoded information for different values of the detuning parameter, which is enough to show the effect of different initial state settings on the dynamics of information. The effect of the detuning parameter on the coded and decoded information is the same as that shown in Fig.(1). From Fig.(4) and Fig.(1), it is clear that coding information in a system prepared initially in an excited state, $|e\rangle$ is much better than using the ground state $|g\rangle$.

From our finding, the users can coded their information with high rate by increasing the detuning parameter or decreasing the number of photons inside the cavity. However the number of oscillations increases for smaller values of the detuning parameter and larger

values of the number of photons. Coding information in a system of Cooper pair prepared initially in an excited state is much better than prepared it in ground state.

4 Quantum coding via imperfect operations

In this section, the dynamics of the coded and decoded information are investigated when some of the local operations are performed imperfectly during the coding process. In this context, we assume that there are two types of imperfect operations: the *first*, Alice performs the bit flip operation with a certain probability, while the *second*, instead of performing bit flip, Alice performs a phase flip operation.

4.1 First type of imperfect operation

Let us assume that during the coding process, Alice applies the operation σ_x successful with probability q and fails with probability $(1 - q)$. In this case, the information is coded in the state,

$$\rho_c = \frac{q}{4}\sigma_{1x}\rho_s(t)\sigma_{1x} + \frac{1-q}{4}\rho_s(t) + \frac{1}{4}\sum_{i=0}^2\left\{\sigma_{1i}\otimes I_2\rho_s(t)\sigma_{1i}\otimes I_2\right\}, \quad (12)$$

where $\sigma_{1i} = I_1, \sigma_{1y}$ and σ_{1z} are the other local operations which are performed perfectly.

Fig.(5), shows the behavior of the coded and decoded information where Alice performed the bit flip operation correctly with probability $q = 0.1$. From Fig(5a), it is clear that the coded information, I_{cod} decreases very fast for small values of the detuning parameter. However as one increases Δ , the coded information decreases gradually and the minimum value is larger than that depicted for small values of Δ . The behavior of the decoded information, I_{Bob} is displayed in Fig.(5b), where the amount of the extracted information increases as one increase Δ . Also, the minimum values of I_{Bob} increases as the detuning parameter increases.

The effect of the number of photons, n inside the cavity is shown in Fig.(6). It is clear that, as one increases n , the coded information decreases. The minimum value is much larger than that depicted in Fig.(5a). This shows that, the amount of information which is coded

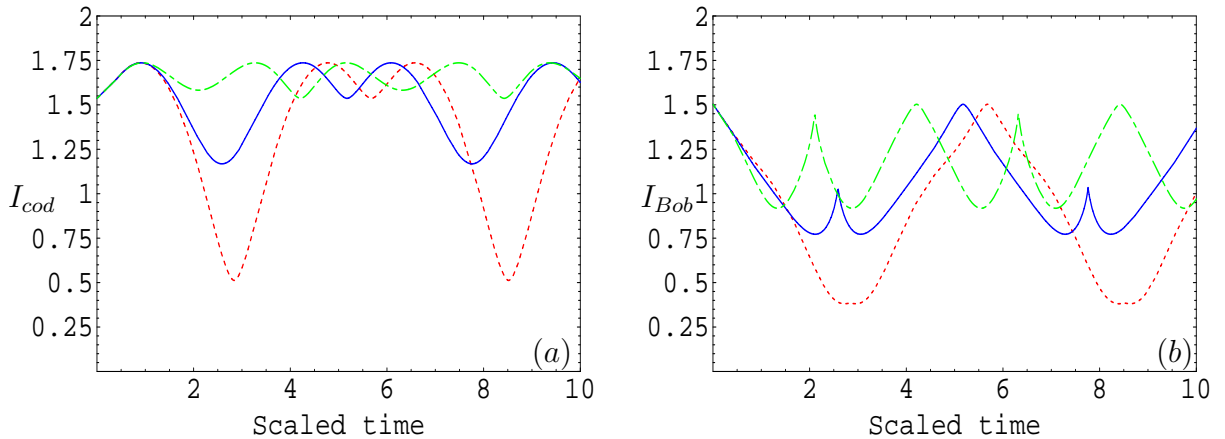


Figure 5: The same as Fig.(1) but Alice performs the bit flip operation with a probability $q = 0.1$ and fails with probability $q = 0.9$ (first type of imperfect operation).

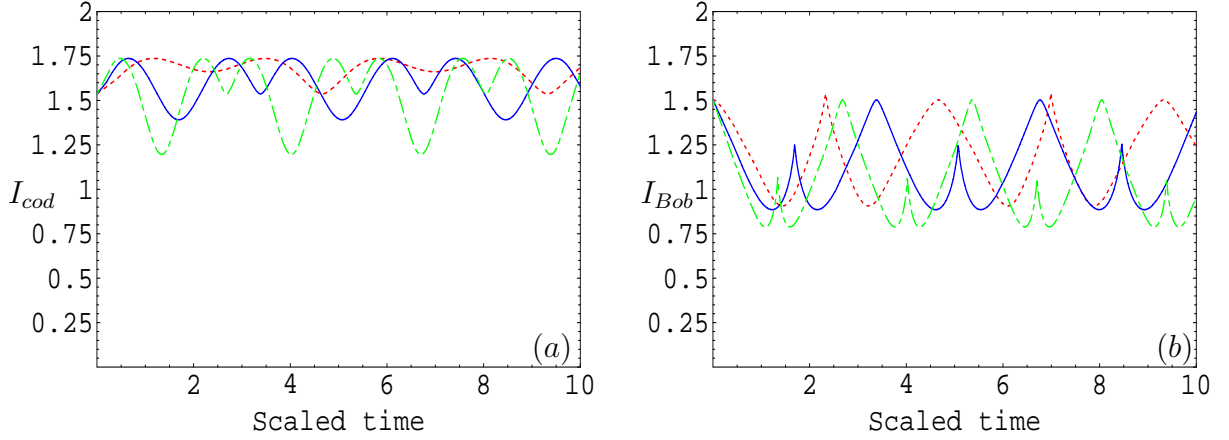


Figure 6: The same as Fig.(5) but for different values of the number of photons inside the cavity. The dot, solid and dash-dot curves $n = 1, 5, 10$ respectively.

in the system (8) can be improved by increasing the detuning parameter or decreasing the number of photons inside the cavity. The behavior of the decoded information is displayed in Fig.(5b). In general the behavior of I_{Bob} is similar to that shown in Fig.(5b). However the number of fluctuations increases as one increases n and the minimum value of the decoded information increases for larger values of n .

4.2 Second type of imperfect operation

We assume that Alice applies the local operation σ_z (phase flip), instead of σ_x (bit flip) during the coding process. In this case, the information is coded in the state,

$$\rho_c = \frac{1}{2}\sigma_{1z} \otimes I_2 \rho_s(t) \sigma_{1z} \otimes I_2 + \frac{1}{4} \sum_{i=0}^1 \left\{ \sigma_{1i} \otimes I_2 \rho_s(t) \sigma_{1i} \otimes I_2 \right\}, \quad (13)$$

where $\sigma_{1i} = I_1, \sigma_{1y}$. The amount of information which is coded in the state (13) is given by

$$I_{cod} = -\frac{3}{4} \left[(c_1^2 \log_2 \frac{3c_1^2}{4} + c_4^2 \log_2 \frac{3c_4^2}{4} + \frac{c_1^2 + c_4^2}{3} \log_2 \frac{c_1^2 + c_4^2}{4} \right] \quad (14)$$

Fig.(7) shows the dynamics of the coded information in the presence of imperfect operation for different values of the detuning parameter Δ , while the other parameters are fixed. From Fig.(7a), it is clear that the general behavior is the similar to that shown in Fig.(5a), but the upper bound is smaller. Fig.(7b) displays the effect of Δ , on the dynamics of the decoded information. In this case, the upper and lower bounds are smaller than that shown in Fig.(6b). The effect of the number of photons, n on the dynamics of the coded and decoded information is described in Fig.(8). In general these phenomena behaves as that shown in Fig.(7), only the number of oscillations increases as n increases.

Figs.(5&6) and Figs.(7&8), the dynamics of information is very sensitive to the noisy operations during the coding process. This shows that the error in achieving the phase bit operation has a larger effect than the error of applying the bit flip operation on the dynamics of information.

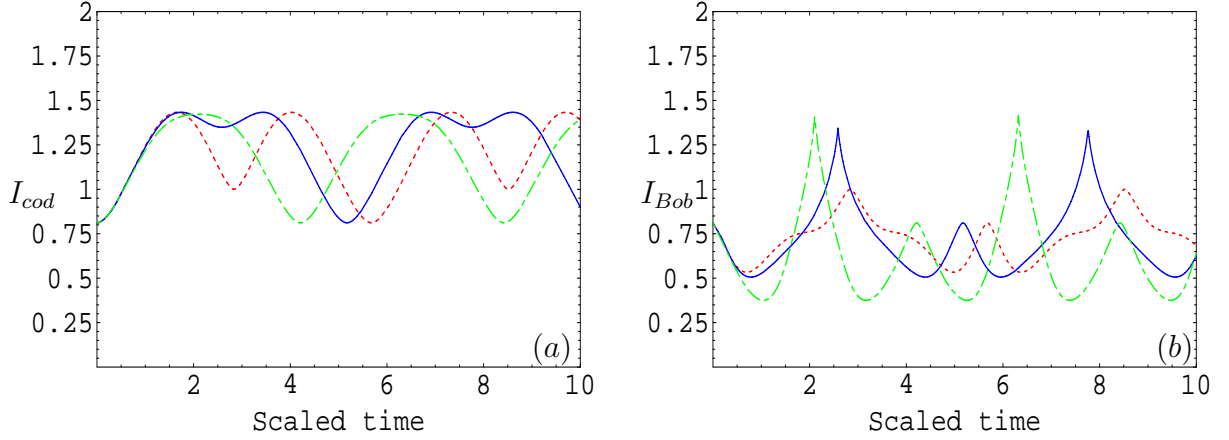


Figure 7: The same as Fig.(1) but the Alice applies phase flip operation instead of the bit flip operation (second type of imperfect operation).

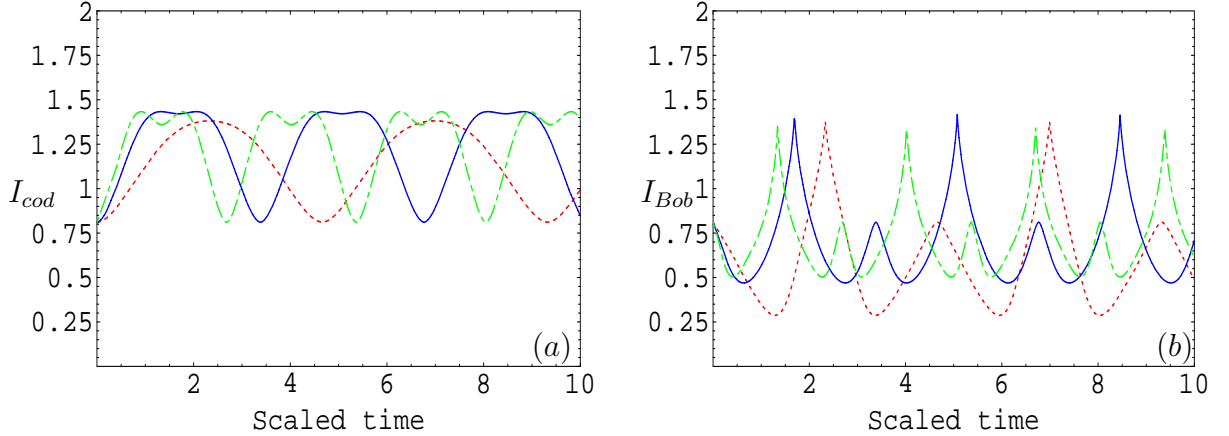


Figure 8: The same as Fig.(2) but Alice applies phase flip operation instead of the bit flip operation (second type of imperfect operation).

5 Disturbance

In this section, we quantify how much the coded information is disturbed or equivalently how much the initial coded and final decoded states are not coincide. This phenomena is called a disturbance of information which is defined as [16, 17],

$$\mathcal{D} = 1 - \mathcal{F}, \quad (15)$$

where \mathcal{F} is the average fidelity. The dynamics of the disturbance is displayed in Fig.(9) for different types of imperfect local operations. It is clear that, when Alice applies the flip operator imperfectly, first type of imperfect operation, (Fig.9a&9b) the upper bound of the disturbance is much smaller than that described for the second type of imperfect operation (Fig.9c&9d). This means that the average fidelity of the decoded information in the presence of the first type of imperfect operation is much better than that for the second type. In other words, the coded information is very sensitive to the phase error.

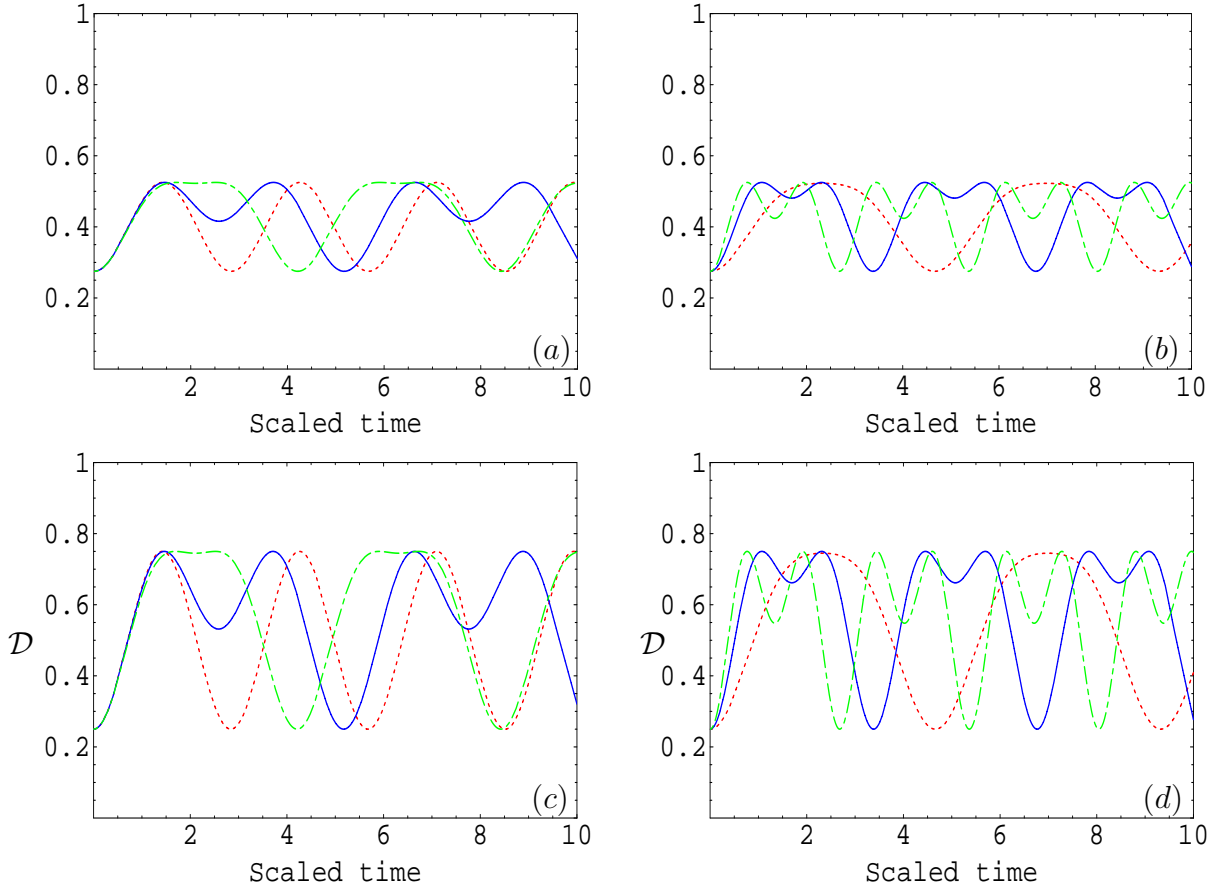


Figure 9: Figs.(a&b) represent the disturbance when the user applies the first type of imperfect operations, while Figs.(c&d) for the second type of imperfect operations. In Figs.(a&c), the dot, solid and dash-dot for $\Delta = 0, 0.5$ and 1 respectively, while in Figs.(b&d) $n = 1, 5$ and 10 respectively.

6 Conclusion

The dynamics of the encrypted information in a single Cooper pair interacts with a cavity is investigated. The effect of the filed and the Cooper pair's parameters on the dynamics of coded and decoded information is discussed. We can see that as soon as the interaction goes on the coded information increases, which means that the generated entangled state between the cavity mode and the Cooper qubit can be used to code information with high efficiency. This efficiency depends on the initial state settings of the Cooper pair, where it is shown that the quantity of coded and decoded information is larger if the Cooper qubit is prepared initially in an excited state. Also, it is shown that the users can improve the amount of the coded and decoded information by increasing the detuning parameter or decreasing the number of photons. However the amount of the decoded information is much smaller than the coded information. Therefore there are some information has been lose during the transforming process.

The dynamics of these phenomena are investigated in the presence of local imperfect operations, where it is assumed that during the coding process one of the local operations is performed in imperfectly. We consider that the Alice applies the bit flip operation with a probability q and files with a probability $(1-q)$. In this case, the decoded information decreases very fast for small values of the detuning parameter, while decreases smoothly for

larger values of photons. However as one increases the detuning parameter the travelling information suppressing the type of the local environmental noise. However decreasing the number of photons inside the cavity can resist the lose of information.

For the other imperfect operation, where the Alice applies the phase flip operation instead of the bit flip operation, the upper bound of the coded and decoded information is much smaller than that depicted for the first type noise. This means that the suppressing of the travelling coded information to this type of noise is fragile. However the lower bound of the decoded information for this type of noise is much smaller than the first type of noise. This shows that the error in achieving the phase bit operation has a larger effect than the error of applying the bit flip operation, namely the coded and decoded information are very sensitive to the phase operations.

Finally, the lost information due to the imperfect operation is quantified. It is shown that the efficiency of the channel between the users decreases, where there is a large disturbance of the information during the coded and decoded processes. However this disturbance due to the error in the achieving the bit flip operation is smaller than that shown for the phase flip error.

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